Tables for Determining Expected Cost Per Unit under MIL-STD-105D Single Sampling Schemes

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Abstract: When a MIL-STD-105D sampling scheme is used for a long period, some lots will be subjected to normal, some to reduced, and some to tightened inspection. This paper provides for several single sampling plans and various quality levels, the expected fraction of lots rejected, the expected sample size per lot, and the expected number of lots to be processed before sampling inspection must be discontinued. Equations are given to calculate the long term cost of sampling inspection using these expected values and appropriate cost parameters.

■ Many private and government purchasers of manufactured products require that each lot submitted be subjected to sampling inspection by attributes. Lots which contain too many defectives may be returned to the manufacturer, purchased with a price concession, subjected to 100% screening, or scrapped. Clearly, there are substantial costs involved for inspection, disposal of rejected lots, and for the occurrence of defectives in accepted lots.

Dodge and Romig (2) have devised a set of attributes sampling plans based upon minimum cost, assuming a desired incoming quality. Hald (5) has greatly enlarged this idea, and developed plans which minimize cost for any prior distribution. However, neither the Dodge-Romig nor the Hald approach have achieved widespread popularity. Attributes sampling in the western world is dominated by the set of plans designated MIL-STD-105D (6) first published by the Department of Defense in 1963.

The MIL-STD-105D plans are not based upon cost concepts. Instead, the plans are indexed by lot size and by a number designated "acceptable quality level." The AQL is specified by the consumer, and is defined as the percent defective which will lead to a high probability of acceptance. This probability of acceptance is not a constant, but varies with lot size and AQL. The domain for probability of acceptance in the MIL-STD-105D plans is about 0.89-0.99.

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Each plan in MIL-STD-105D provides a sample size, n, and an acceptance number, c, to be used for "normal" inspection of a lot. If c, or fewer defectives are found in the sample, the lot is accepted. The user is required to keep a historical record of lot-by-lot experience. Criteria are presented for an alteration of the values of n and/or c when the experience over several lots shows either unusually good or unusually bad quality. The rules are as follows (6):

1. A switch from the normal values of n and c to "reduced" inspection is permissible when
   a. Ten consecutive lots have been accepted.
   b. The total number of defectives in the ten lots does not exceed a critical value supplied in Table VIII of MIL-STD-105D.
   c. Production is continuous.
   d. Reduced inspection is considered desirable by the responsible authority.

Under reduced inspection, n is substantially decreased to a value, n_R. Two numbers, c and $r(>c)$ are supplied. Lots are accepted if the number of defectives is less than r. However, if a lot has more than c defectives, normal inspection must be resumed on the next lot.

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When a MIL-STD-IOSD sampling scheme is used for a long period, some lots will be subjected to normal, some to reduced, and some to tightened inspection. This paper provides for several single sampling plans and various quality levels,’the expected fraction of lots rejected,the expected sample size per lot, and the expected number of lots to be processed before sampling inspection must be discontinued. Eq.uations are given to calculate the long term cost of sampling inspection using these expected values and appropriate cost parameters.

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2. A switch to "tightened" from normal inspection is required when two of the most recent five lots have been rejected. Under tightened inspection, the sample size, n_T, is usually the same as for normal inspection, but c is reduced. A return to normal is permitted when five consecutive lots have been accepted. If tightened, inspection is still in use for ten consecutive lots; however, sampling inspection must be discontinued entirely.

It is clear that, if many lots from a process producing a fraction defective, p, are submitted under such a scheme, some lots will be subjected to normal inspection, some to reduced, and some to tightened inspection. If the probability of acceptance under reduced, normal, and tightened inspection is designated $P_{A,R}$, $P_{A,N}$, $P_{A,T}$ respectively, then this probability is progressively lower as we go from reduced to normal to tightened, so the fraction of lots rejected will depend upon the proportion inspected under each of the three plans. Furthermore, unless p is zero, it is inevitable that eventually, during one of the adoptions of tightened inspection, the criterion for return to normal inspection will not be met during the next 10 lots, so that sampling inspection ultimately will be abandoned.

The subject of this paper is the cost of lot-by-lot sampling inspection under the MIL-STD-105D plans. It is traditional in the literature of quality control to examine the performance of an attributes sampling plan under the assumption that, when the process is "in control," a stream of product is being produced with a fixed probability, p, that each item is defective [see Duncan (3), p. 147 or Grant and Leavenworth (4) p. 364]. The value of p for a particular kind of manufacturing process is usually well established. We may think of p as a parameter of a production process in control, a characteristic of the process. The purpose of attributes sampling inspection is, of course, to guard against sudden, "out of control" increases in p. However, if p is constant, and lots are formed and inspected under an attributes plan, there is a nonzero probability that each lot will be rejected, even though rejected and accepted lots have the same underlying quality. This "producer's risk" is an inherent consequence of acceptance sampling by attributes. We propose to answer the following question in the next section of this paper. If a manufacturer can consistently maintain a quality level, p, what will be the expected cost incurred per manufactured item as a result of exposure to MIL-STD-105D attributes sampling? In the third section, we explore the case where p is unknown, but where upper and lower bounds for p are available.

Brown and Rutemiller (1) have formulated a mathematical model of the MIL-STD-105D sampling scheme treating normal, reduced and tightened inspection as three stochastically coupled Markov chains. Using this analysis one may obtain the following information for any (lot size, AQL, p) combination:

f_N = Expected fraction of lots under normal inspection;

f_R = Expected fraction of lots under reduced inspection;

f_T = Expected fraction of lots under tightened inspection;

L = Expected number of lots inspected before sampling inspection must be abandoned;

f = Expected fraction of lots rejected during sampling inspection.

The table results in the present paper were obtained using this model.

Assumptions

There are several costs which must be known or estimated to determine the total cost of sampling inspection under MIL-STD-105D.

Let

k_1 = cost of inspecting a single item under sampling inspection.

k_2 = cost of inspecting a single item under 100% screening inspection;

k_3 = cost of replacement for a single defective item detected under either sampling inspection or 100% screening;

k_4 = cost of replacement for a single defective item detected later in the manufacturing process;

k_5 = cost of discontinuing sampling inspection completely.

The costs, k_1 and k_2, will frequently differ since k_1 includes the cost of gathering a random sample. In addition, economies of scale occur when an entire lot is inspected.

The cost, k_3, will often be substantially lower than k_4 because additional labor may be expended on items in accepted lots; when one of these is subsequently found to be defective, such additional labor costs are not recoverable. In addition, k_4 may include the cost of damage to a finished product of which the item is a component, customer reaction to a defective product, etc.

The cost, k_5, which occurs when sampling inspection is discontinued because too many consecutive lots have been on tightened inspection, will be generated by whatever remedial action is required to again institute sampling inspection. For example, this could involve a stopping of production for adjustments, frequently accompanied by a requirement that the next L lots be subjected to 100% screening before sampling inspection is resumed.

We define a *cycle* as the expected number of lots which will be subjected to sampling inspection until the tightened inspection rules of MIL-STD-105D require discontinuation of sampling inspection. For any (lot size, AQL, p) combination we define

N = lot size;

$\bar{n}$ = expected sample size during sampling inspection;

T = expected number of lots under sampling inspection during one *cycle*;

C_I = expected cost per manufactured item incurred from MIL-STD-105D sampling.

We have

$$\bar{n} = f_N \cdot n + f_T \cdot n_T + f_R \cdot n_R \; ;$$

$$f = 1 - [f_N \cdot P_{A,N} + f_T \cdot P_{A,T} + f_R \cdot P_{A,R}] \; .$$

The value of T and the parameters needed to calculate $\bar{n}$ and f may be obtained from solution of the Markov matrices (1, p. 194).

The total cost incurred during one *cycle* will be

$$C = T\{k_1\bar{n} + k_2(N-\bar{n})f + k_3[\bar{n} + (N-\bar{n})f]p + k_4(N-\bar{n})(1-f)p\} + k_5 \; .$$

The number of items manufactured will be $N(T+L)$. Hence,

$$C_I = \frac{C}{N(T+L)}$$

Discussion

The information in Table 1 will provide, for a particular lot size, AQL, and p, an estimate of the cost per unit attributable to defective items, when a "stream of product," each item having a probability, p, of being defective, is formed into lots. Several extensions of these calculations are possible.

If we have an accurate estimate of p, say from previous experience on similar products, then C_I may be computed for several of the AQL plans in *MIL-STD-105D* to find the sampling plan yielding minimum cost per unit. In many instances, lot size may also be set by the manufacturer. In this case, we could examine the various (AQL, lot size) combinations in MIL-STD-105D to ascertain the minimum cost combination.

An estimated *domain* for p may be available from prior experience on similar products. Clearly, it will be useful to employ the upper bound of this domain in conjunction with Table 1 to obtain an upper bound for costs attributable to sampling inspection. Even if p proves to be a random variable from lot to lot, costs from sampling will not exceed those calculated under "worst case" assumption that all lots are at the upper bound.

If p is completely unknown, we may still obtain valuable information from Table 1. For example, we can determine what quality level, p, must be maintained in production to hold the cost per item attributable to sampling inspection to X dollars, where X is a break-even value, or a value necessary to maintain a minimum profit level. Often, a knowledge of this *required* p is sufficient to ascertain whether a particular production method is practicable.

Examples

(1) MIL-STD-105D is to be instituted on lots of size 100, using general inspection level II, AQL = 4%. It is expected that the AQL level will be maintained in production. The cost of inspecting a single item under sampling is estimated to be \$1.80. Rejected lots are to be 100% inspected, and the inspection cost is estimated at \$1.20 per item during 100% screening. Each defective item costs \$10.00 to replace if discovered during sampling or screening inspection. If discovered later in the production process, the cost is estimated at \$30.00. When sampling is discontinued, the cost incurred is estimated at \$600.00, including the cost of inspecting ten lots.

We have

$$\begin{aligned} N &= 100; \\ \text{AQL} &= 4.0; \\ 100p &= 4.0; \\ k_1 &= \$\,1.80; \\ k_2 &= \$\,1.20; \\ k_3 &= \$10.00; \\ k_4 &= \$30.00; \\ k_5 &= \$600.00; \\ L &= 10 \, . \end{aligned}$$

From Table 1,

$$\begin{aligned} \bar{n} &= 16.3; \\ 100f &= 3.9; \\ T &= 661 . \end{aligned}$$

Therefore,

$$C = 661\{1.80(16.3) + 1.20(100-16.3)(.039) + 10.00[16.3 + (100-16.3)(.039)](.04) + 30(100-16.3)(1-.039)(.04)\} + 600 = 91{,}557.38;$$

$$C_I = 91{,}557.38/100(661+10)$$

$$= \$1.36.$$

(2) Suppose that 100% inspection is used for this process at all times, in lieu of MIL-STD-105D inspection..

Then

$$C_I = 1.20 + 10(.04)$$

$$= \$1.60 \, .$$

So that sampling inspection saves about \$0.23 per manufactured item in this case.

(3) Find the AQL plan which will minimize the cost of defective items for this process.

Using information from Table 1, and linear interpolation, we get

AQL	$\bar{n}$	$100f$	T	C_I
1.0	17.6	50.0	16	\$1.18
1.5	43.2	50.7	17	1.32
2.5	24.4	25.7	34	1.31
4.0	16.3	3.9	661	1.36
6.5	12.9	0.6	2.55×10^6	1.33
10.0	8.7	0.0	6.60×10^9	1.29

Table 1: Expected sample size, expected percentage of lots rejected, and expected number of lots before Discontinuation of sampling inspection for some representative MIL-STD-105D single-sampling plans and quality levels.

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling	AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling
0.015	Q	0.00375	827.8	3.1	3,309	0.25	K-L	0.0625	121.4	0.3	3.49×10^6
		0.0075	856.4	6.2	535			0.125	150.1	1.8	15,686
		0.01125	884.6	9.5	203			0.1875	176.7	4.7	982
		0.015	911.5	12.7	109			0.25	201.0	9.4	194
		0.01875	936.4	16.0	70			0.3125	221.5	15.6	73
		0.0225	959.1	19.3	51			0.375	237.5	22.5	40
		0.02625	979.6	22.5	40			0.4375	249.6	29.7	28
		0.03	997.7	25.7	33			0.5	258.6	36.7	22
		0.03375	1,013.9	28.8	28			0.5625	265.3	43.4	19
		0.0375	1,028.1	31.7	25			0.625	270.5	49.6	17
		0.04125	1,040.7	34.6	23			0.6875	274.6	55.3	16
		0.045	1,051.8	37.4	21			0.75	277.8	60.5	15
		0.04875	1,061.6	40.0	19			0.8125	280.4	65.2	14
		0.0525	1,070.3	42.6	18			0.875	282.5	69.4	14
		0.05625	1,078.1	45.0	18			0.9375	284.3	73.1	14
		0.06	1,085.0	47.4	17			1.0	285.7	76.4	13
0.04	M	0.01	327.0	3.2	2,814	0.25	M	0.0625	130.4	0.0	1.49×10^9
		0.02	339.4	6.6	462			0.125	148.1	0.1	800,001
		0.03	351.6	10.0	178			0.1875	192.4	0.9	10,379
		0.04	363.1	13.5	97			0.25	251.8	3.6	753
		0.05	373.7	17.0	63			0.3125	292.9	8.9	158
		0.06	383.2	20.4	46			0.375	309.3	16.0	63
		0.07	391.7	23.8	36			0.4375	313.8	24.0	36
		0.08	399.2	27.1	30			0.5	314.8	32.1	26
		0.09	405.8	30.4	26			0.5625	315.0	39.9	21
		0.10	411.6	33.5	23			0.625	315.0	47.1	18
		0.11	416.6	36.4	21			0.6875	315.0	53.6	16
		0.12	421.1	39.3	20			0.75	315.0	59.5	15
		0.13	425.0	42.0	19			0.8125	315.0	64.7	15
		0.14	428.5	44.7	18			0.875	315.0	69.3	14
		0.15	431.5	47.2	17			0.9375	315.0	73.4	14
		0.16	434.3	49.6	16			1.0	315.0	76.9	13
0.065	L-M	0.01625	207.7	3.3	2,622	0.40	G-H	0.1	33.2	3.3	2,774
		0.0325	215.6	6.8	433			0.2	34.4	6.6	456
		0.4875	223.4	10.3	168			0.3	35.6	10.1	176
		0.065	230.7	13.9	92			0.4	36.7	13.6	95
		0.08125	237.4	17.5	60			0.5	37.8	17.1	62
		0.0975	243.4	21.0	44			0.6	38.7	20.6	46
		0.11375	248.7	24.5	35			0.7	39.5	24.0	36
		0.13	253.4	27.9	29			0.8	40.3	27.4	30
		0.14625	257.5	31.1	26			0.9	40.9	30.6	26
		0.1625	261.0	34.3	23			1.0	41.5	33.7	23
		0.17875	264.2	37.3	21			1.1	42.0	36.7	21
		0.195	266.9	40.2	19			1.2	42.4	39.5	20
		0.21125	269.3	43.0	18			1.3	42.8	42.3	19
		0.2275	271.4	45.6	17			1.4	43.1	44.9	18
		0.24375	273.3	48.1	17			1.5	43.4	47.4	17
		0.26	274.9	50.6	16			1.6	43.7	49.8	16
0.15	J-K	0.0375	82.8	3.1	3,309	0.40	J-K	0.1	75.9	0.3	3.29×10^6
		0.075	85.6	6.2	535			0.2	93.9	1.8	14,916
		0.1125	88.5	9.5	203			0.3	110.6	4.7	941
		0.15	91.1	12.7	109			0.4	126.1	9.5	188
		0.1875	93.6	16.0	70			0.5	139.3	15.7	71
		0.225	95.9	19.3	51			0.6	149.6	22.8	39
		0.2625	98.0	22.5	40			0.7	157.4	30.1	27
		0.3	99.8	25.7	33			0.8	163.2	37.2	22
		0.3375	101.4	28.8	28			0.9	167.6	43.9	19
		0.375	102.8	31.7	25			1.0	171.0	50.2	17
		0.4125	104.1	34.6	23			1.1	173.6	55.9	16
		0.45	105.2	37.4	21			1.2	175.7	61.1	15
		0.4875	106.2	40.0	19			1.3	177.4	65.8	14
		0.525	107.0	42.6	18			1.4	178.8	69.9	14
		0.5625	107.8	45.0	18			1.5	179.9	73.6	14
		0.6	108.5	47.4	17			1.6	180.9	76.9	13
0.25	H-J	0.0625	51.9	3.2	2,835	0.40	L	0.1	83.6	0.0	1.42×10^9
		0.125	53.9	6.5	465			0.2	95.3	0.2	664,173
		0.1875	55.9	9.9	179			0.3	124.7	1.0	8,789
		0.25	57.8	13.4	97			0.4	162.6	3.9	661
		0.3125	59.5	16.9	63			0.5	187.5	9.4	144
		0.375	61.0	20.3	46			0.6	196.9	16.8	59
		0.4375	62.4	23.7	37			0.7	199.4	24.9	35
		0.5	63.6	27.0	31			0.8	199.9	33.1	25
		0.5625	64.7	30.2	26			0.9	200.0	40.9	20
		0.625	65.6	33.3	24			1.0	200.0	48.2	18
		0.6875	66.4	36.3	21			1.1	200.0	54.7	16
		0.75	67.1	39.2	20			1.2	200.0	60.5	15
		0.8125	67.8	41.9	19			1.3	200.0	65.7	14
		0.875	68.3	44.6	18			1.4	200.0	70.2	14
		0.9375	68.8	47.1	17			1.5	200.0	74.2	14
		1.0	69.3	49.5	16			1.6	200.0	77.7	13

Table 1 (continued)

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling
0.40	M	0.1	137.8	0.0	2.50×10^{10}
		0.2	167.5	0.1	7.34×10^{6}
		0.3	231.4	0.9	30,529
		0.4	283.8	3.6	1,303
		0.5	307.1	8.7	198
		0.6	313.7	16.3	65
		0.7	314.8	25.5	34
		0.8	315.0	35.1	23
		0.9	315.0	44.4	18
		1.0	315.0	52.9	16
		1.1	315.0	60.4	15
		1.2	315.0	67.0	14
		1.3	315.0	72.6	14
		1.4	315.0	77.5	13
		1.5	315.0	81.5	13
		1.6	315.0	84.9	13
0.65	F-G	0.1625	20.8	3.3	2,548
		0.325	21.6	6.8	422
		0.4875	22.5	10.3	164
		0.65	23.2	14.0	90
		0.8125	23.9	17.6	59
		0.975	24.5	21.2	44
		1.1375	25.1	24.7	35
		1.3	25.6	28.1	29
		1.4625	26.0	31.4	25
		1.625	26.4	34.5	23
		1.7875	26.7	37.6	21
		1.95	27.0	40.5	19
		2.1125	27.2	43.3	18
		2.275	27.5	46.0	17
		2.4375	27.6	48.5	17
		2.6	27.8	50.9	16
0.65	H-J	0.1625	49.1	0.4	2.61×10^{6}
		0.325	60.9	1.9	12,090
		0.4875	71.9	5.2	787
		0.65	81.7	10.3	163
		0.8125	89.8	16.8	64
		0.975	95.9	24.1	37
		1.1375	100.5	31.5	26
		1.3	103.8	38.7	21
		1.4625	106.3	45.5	18
		1.625	108.3	51.7	16
		1.7875	109.8	57.4	15
		1.95	110.9	62.5	15
		2.1125	111.9	67.1	14
		2.275	112.7	71.2	14
		2.4375	113.3	74.9	14
		2.6	113.9	78.1	13
0.65	K	0.1625	52.3	0.0	1.11×10^{9}
		0.325	60.0	0.2	560,102
		0.4875	79.1	1.1	7,541
		0.65	102.9	4.2	586
		0.8125	117.9	9.9	132
		0.975	123.3	17.5	56
		1.1375	124.7	25.8	33
		1.3	124.9	34.1	24
		1.4625	125.0	42.0	20
		1.625	125.0	49.2	17
		1.7875	125.0	55.7	16
		1.95	125.0	61.5	15
		2.1125	125.0	66.7	14
		2.275	125.0	71.2	14
		2.4375	125.0	75.1	13
		2.6	125.0	78.6	13
1.0	E-F	0.25	13.5	3.3	2,735
		0.5	13.9	6.7	450
		0.75	14.4	10.2	173
		1.0	14.9	13.8	94
		1.25	15.3	17.3	62
		1.5	15.6	20.8	45
		1.75	16.0	24.2	36
		2.0	16.2	27.6	30
		2.25	16.5	30.8	26
		2.5	16.7	33.9	23
		2.75	16.9	36.9	21
		3.0	17.1	39.7	20
		3.25	17.2	42.5	18
		3.5	17.3	45.1	17
		3.75	17.5	47.6	17
		4.0	17.6	50.0	16
1.0	G-H	0.25	30.4	0.3	3.30×10^{6}
		0.5	37.5	1.8	14,916
		0.75	44.2	4.7	941
		1.0	50.4	9.5	188
		1.25	55.7	15.7	71
		1.5	59.9	22.8	39
		1.75	63.0	30.1	27
		2.0	65.3	37.2	22
		2.25	67.1	43.9	19
		2.5	68.4	50.2	17
		2.75	69.5	55.9	16
		3.0	70.3	61.1	15
		3.25	71.0	65.8	14
		3.5	71.5	69.9	14
		3.75	72.0	73.6	14
		4.0	72.4	76.9	13
1.0	J	0.25	33.4	0.0	1.42×10^{9}
		0.5	38.1	0.2	664,173
		0.75	49.9	1.0	8,789
		1.0	65.0	3.9	661
		1.25	75.0	9.4	144
		1.5	78.8	16.8	59
		1.75	79.8	24.9	35
		2.0	80.0	33.1	25
		2.25	80.0	40.9	20
		2.5	80.0	48.2	18
		2.75	80.0	54.7	16
		3.0	80.0	60.5	15
		3.25	80.0	65.7	14
		3.5	80.0	70.2	14
		3.75	80.0	74.2	14
		4.0	80.0	77.7	13
1.0	K	0.25	55.0	0.0	2.50×10^{9}
		0.5	68.0	0.1	7.00×10^{6}
		0.75	98.8	1.0	28,432
		1.0	116.7	3.7	1,314
		1.25	123.2	8.6	205
		1.5	124.7	15.9	67
		1.75	125.0	25.0	35
		2.0	125.0	34.6	23
		2.25	125.0	43.8	19
		2.5	125.0	52.2	16
		2.75	125.0	59.8	15
		3.0	125.0	66.4	14
		3.25	125.0	72.1	14
		3.5	125.0	77.0	13
		3.75	125.0	81.1	13
		4.0	125.0	84.5	13
1.5	D-E	0.375	8.3	3.1	3,080
		0.75	8.6	6.3	507
		1.125	8.9	9.6	192
		1.5	9.2	12.9	103
		1.875	9.5	16.3	67
		2.25	9.8	19.6	49
		2.625	10.0	22.9	38
		3.0	10.2	26.2	32
		3.375	10.4	29.3	27
		3.75	10.5	32.4	24
		4.125	10.7	35.3	22
		4.5	10.8	38.1	20
		4.875	10.9	40.8	19
		5.25	11.0	43.4	18
		5.625	11.1	45.9	17
		6.0	11.2	48.3	17
1.5	F-G	0.375	20.0	0.3	4.45×10^{6}
		0.75	25.5	1.8	186,217
		1.125	29.8	4.7	1,145
		1.5	33.0	9.1	226
		1.875	35.5	14.6	84
		2.25	37.7	20.9	45
		2.625	39.4	27.7	30
		3.0	40.8	34.4	23
		3.375	41.9	40.9	20
		3.75	42.7	47.0	18
		4.125	43.4	52.6	16
		4.5	43.9	57.8	15
		4.875	44.4	62.5	15
		5.25	44.7	66.8	14
		5.625	45.0	70.6	14
		6.0	45.3	74.1	14

Table 1 (continued)

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling
1.5	H	0.375	20.8	0.0	2.15×10^9
		0.75	24.0	0.1	1.07×10^6
		1.125	32.8	1.0	12,457
		1.5	42.4	3.5	922
		1.875	47.7	8.0	196
		2.25	49.4	14.2	76
		2.625	49.9	21.4	42
		3.0	50.0	29.1	29
		3.375	50.0	36.6	23
		3.75	50.0	43.7	19
		4.125	50.0	50.3	17
		4.5	50.0	56.2	16
		4.875	50.0	61.5	15
		5.25	50.0	66.3	14
		5.625	50.0	70.5	14
		6.0	50.0	74.2	14
1.5	J	0.375	35.0	0.0	2.69×10^9
		0.75	42.5	0.1	1.32×10^7
		1.125	61.2	0.9	46,481
		1.5	73.4	3.1	1,946
		1.875	78.4	7.4	277
		2.25	79.7	14.0	84
		2.625	80.0	22.3	40
		3.0	80.0	31.5	26
		3.375	80.0	40.5	20
		3.75	80.0	49.0	17
		4.125	80.0	56.6	15
		4.5	80.0	63.3	14
		4.875	80.0	69.2	14
		5.25	80.0	74.3	13
		5.625	80.0	78.7	13
		6.0	80.0	82.4	13
1.5	K	0.375	50.7	0.0	1.77×10^{10}
		0.75	54.8	0.0	2.69×10^9
		1.125	85.8	0.2	996,132
		1.5	114.3	1.1	12,153
		1.875	122.8	3.6	797
		2.25	124.7	9.1	144
		2.625	125.0	18.4	51
		3.0	125.0	30.5	29
		3.375	125.0	42.9	21
		3.75	125.0	54.2	17
		4.125	125.0	63.8	15
		4.5	125.0	71.7	14
		4.875	125.0	78.1	14
		5.25	125.0	83.2	13
		5.625	125.0	87.2	13
		6.0	125.0	90.3	13
2.5	C-D	0.625	5.2	3.2	2,835
		1.25	5.4	6.5	465
		1.875	5.6	9.9	179
		2.5	5.8	13.4	97
		3.125	5.9	16.9	63
		3.75	6.1	20.3	46
		4.375	6.2	23.7	37
		5.0	6.4	27.0	31
		5.625	6.5	30.2	26
		6.25	6.6	33.3	24
		6.875	6.6	36.3	21
		7.5	6.7	39.2	20
		8.125	6.8	41.9	19
		8.75	6.8	44.6	18
		9.375	6.9	47.1	17
		10.0	6.9	49.5	16
2.5	E-F	0.625	12.1	0.3	3.30×10^6
		1.25	15.0	1.8	14,916
		1.875	17.7	4.7	941
		2.5	20.2	9.5	188
		3.125	22.3	15.7	71
		3.75	23.9	22.8	39
		4.375	25.2	30.1	27
		5.0	26.1	37.2	22
		5.625	26.8	43.9	19
		6.25	27.4	50.2	17
		6.875	27.8	55.9	16
		7.5	28.1	61.1	15
		8.125	28.4	65.8	14
		8.75	28.6	69.9	14
		9.375	28.8	73.6	14
		10.0	28.9	76.9	13
2.5	G	0.625	13.6	0.0	1.38×10^9
		1.25	15.5	0.2	647,412
		1.875	20.2	1.1	8,633
		2.5	26.2	3.9	656
		3.125	30.1	9.4	143
		3.75	31.5	16.8	59
		4.375	31.9	24.9	35
		5.0	32.0	33.1	25
		5.625	32.0	40.9	20
		6.25	32.0	48.2	18
		6.875	32.0	54.7	16
		7.5	32.0	60.5	15
		8.125	32.0	65.7	14
		8.75	32.0	70.2	14
		9.375	32.0	74.3	14
		10.0	32.0	77.7	13
2.5	H	0.625	22.0	0.0	2.50×10^9
		1.25	27.2	0.1	7.00×10^6
		1.875	39.5	1.0	28,432
		2.5	46.7	3.7	1,314
		3.125	49.3	8.6	205
		3.75	49.9	15.9	67
		4.375	50.0	25.0	35
		5.0	50.0	34.5	23
		5.625	50.0	43.8	19
		6.25	50.0	52.2	16
		6.875	50.0	59.8	15
		7.5	50.0	66.4	14
		8.125	50.0	72.1	14
		8.75	50.0	77.0	13
		9.375	50.0	81.1	13
		10.0	50.0	84.5	13
2.5	J	0.625	32.5	0.0	1.48×10^{10}
		1.25	35.6	0.0	1.14×10^9
		1.875	55.0	0.2	410,188
		2.5	73.1	1.5	5,454
		3.125	78.7	5.0	415
		3.75	79.9	12.3	89
		4.375	80.0	23.9	37
		5.0	80.0	37.2	23
		5.625	80.0	49.9	18
		6.25	80.0	60.8	16
		6.875	80.0	69.8	14
		7.5	80.0	77.0	14
		8.125	80.0	82.6	13
		8.75	80.0	86.9	13
		9.375	80.0	90.3	13
		10.0	80.0	92.8	12
2.5	K	0.625	50.2	0.0	5.43×10^{10}
		1.25	52.8	0.0	4.50×10^9
		1.875	73.5	0.1	4.95×10^6
		2.5	113.6	1.4	12,909
		3.125	123.8	7.3	459
		3.75	125.0	14.6	72
		4.375	125.0	29.6	29
		5.0	125.0	45.8	19
		5.625	125.0	60.0	15
		6.25	125.0	71.4	14
		6.875	125.0	80.1	13
		7.5	125.0	86.4	13
		8.125	125.0	90.0	13
		8.75	125.0	94.0	12
		9.375	125.0	96.1	12
		10.0	125.0	97.5	12
4.0	B-C	1.0	3.1	3.1	2,942
		2.0	3.3	6.3	481
		3.0	3.4	9.6	185
		4.0	3.5	13.0	100
		5.0	3.6	16.4	65
		6.0	3.7	19.8	48
		7.0	3.8	23.2	38
		8.0	3.9	26.5	31
		9.0	4.0	29.7	27
		10.0	4.0	32.8	24
		11.0	4.1	35.7	22
		12.0	4.1	38.6	20
		13.0	4.2	41.3	19
		14.0	4.2	44.0	18
		15.0	4.2	46.5	17
		16.0	4.3	48.9	17

Table 1 (continued)

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling	AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling
4.0	D-E	1.0	7.8	0.3	2.84×10^6	6.5	C-D	1.625	4.7	0.3	2.36×10^6
		2.0	9.7	1.9	12,897			3.25	5.9	1.9	10,958
		3.0	11.6	5.1	827			4.875	7.1	5.2	720
		4.0	13.2	10.2	169			6.5	8.2	10.4	151
		5.0	14.5	16.6	66			8.125	9.1	17.2	60
		6.0	15.5	23.8	37			9.75	9.8	24.8	35
		7.0	16.2	31.1	26			11.375	10.3	32.4	25
		8.0	16.7	38.3	21			13.0	10.7	39.8	20
		9.0	17.1	50.0	18			14.625	10.9	46.7	18
		10.0	17.4	51.2	16			16.25	11.1	53.1	16
		11.0	17.6	56.9	15			17.875	11.3	58.8	15
		12.0	17.8	62.0	15			19.5	11.4	63.9	15
		13.0	18.0	66.6	14			21.125	11.5	68.5	14
		14.0	18.1	70.7	14			22.75	11.6	72.6	14
		15.0	18.2	74.4	14			24.375	11.7	76.1	14
		16.0	18.3	77.6	13			26.0	11.8	79.3	13
4.0	F	1.0	8.4	0.0	1.42×10^9	6.5	E	1.625	5.2	0.0	7.91×10^8
		2.0	9.5	0.2	664,173			3.25	6.1	0.2	387,112
		3.0	12.5	1.0	8,789			4.875	8.3	1.3	5,370
		4.0	16.3	3.9	661			6.5	10.9	4.8	443
		5.0	18.8	9.4	144			8.125	12.4	11.3	107
		6.0	19.7	16.8	59			9.75	12.9	19.4	48
		7.0	19.9	24.9	35			11.375	13.0	28.3	30
		8.0	20.0	33.1	25			13.0	13.0	36.7	23
		9.0	20.0	40.9	20			14.625	13.0	44.7	19
		10.0	20.0	48.2	18			16.25	13.0	51.9	17
		11.0	20.0	54.5	16			17.875	13.0	58.4	15
		12.0	20.0	60.5	15			19.5	13.0	64.1	15
		13.0	20.0	65.7	14			21.125	13.0	69.1	14
		14.0	20.0	70.2	14			22.75	13.0	73.4	14
		15.0	20.0	74.2	14			24.375	13.0	77.2	13
		16.0	20.0	77.7	13			26.0	13.0	80.5	13
4.0	G	1.0	14.4	0.0	2.33×10^9	6.5	F	1.625	8.9	0.0	2.33×10^9
		2.0	17.5	0.1	5.72×10^6			3.25	10.9	0.1	4.72×10^6
		3.0	24.1	1.0	24,785			4.875	15.2	1.1	20,813
		4.0	29.2	3.8	1,114			6.5	18.3	4.1	967
		5.0	31.3	9.2	177			8.125	19.6	9.7	159
		6.0	31.9	17.1	60			9.75	19.9	17.9	56
		7.0	32.0	26.6	32			11.375	20.0	27.6	30
		8.0	32.0	36.4	22			13.0	20.0	37.6	22
		9.0	32.0	45.7	18			14.625	20.0	46.9	18
		10.0	32.0	54.1	16			16.25	20.0	55.4	16
		11.0	32.0	61.6	15			17.875	20.0	62.8	14
		12.0	32.0	68.1	14			19.5	20.0	69.2	14
		13.0	32.0	73.7	13			21.125	20.0	74.7	13
		14.0	32.0	78.4	13			22.75	20.0	79.3	13
		15.0	32.0	82.4	13			24.375	20.0	83.2	13
		16.0	32.0	85.7	13			26.0	20.0	86.4	13
4.0	H	1.0	20.3	0.0	1.48×10^{10}	6.5	G	1.625	13.3	0.0	1.27×10^{10}
		2.0	22.3	0.0	1.14×10^9			3.25	14.6	0.0	6.66×10^8
		3.0	34.4	0.2	410,188			4.875	23.6	0.3	207,269
		4.0	45.7	1.5	5,454			6.5	29.9	1.9	3,286
		5.0	49.2	5.0	415			8.125	31.7	6.1	285
		6.0	49.9	12.3	89			9.75	32.0	14.8	69
		7.0	50.0	23.9	37			11.375	32.0	27.6	32
		8.0	50.0	37.2	23			13.0	32.0	41.4	21
		9.0	50.0	49.9	18			14.625	32.0	54.1	17
		10.0	50.0	60.8	16			16.25	32.0	64.6	15
		11.0	50.0	69.8	14			17.875	32.0	73.1	14
		12.0	50.0	77.0	14			19.5	32.0	79.8	13
		13.0	50.0	82.6	13			21.125	32.0	85.0	13
		14.0	50.0	86.9	13			22.65	32.0	88.9	13
		15.0	50.0	90.3	13			24.375	32.0	91.9	13
		16.0	50.0	92.8	12			26.0	32.0	94.1	12
4.0	J	1.0	32.2	0.0	4.97×10^{10}	6.5	H	1.625	20.1	0.0	4.69×10^{10}
		2.0	34.0	0.0	4.15×10^9			3.25	21.3	0.0	3.94×10^9
		3.0	49.8	0.2	2.74×10^6			4.875	29.7	0.2	2.40×10^6
		4.0	73.9	1.5	8,612			6.5	45.4	1.6	6,944
		5.0	79.5	6.0	344			8.125	49.6	6.6	289
		6.0	80.0	16.5	60			9.75	50.0	17.8	54
		7.0	80.0	32.4	26			11.375	50.0	34.2	25
		8.0	80.0	48.7	18			13.0	50.0	50.6	17
		9.0	80.0	62.7	15			14.625	50.0	64.5	15
		10.0	80.0	73.8	14			16.25	50.0	75.2	14
		11.0	80.0	82.0	13			17.875	50.0	83.1	13
		12.0	80.0	87.9	13			19.5	50.0	88.8	13
		13.0	80.0	92.0	12			21.125	50.0	92.7	12
		14.0	80.0	94.8	12			22.75	50.0	95.3	12
		15.0	80.0	96.7	12			24.375	50.0	97.1	12
		16.0	80.0	97.9	12			26.0	50.0	98.2	12

Table 1 (continued)

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling
10.0	C	2.5	3.0	0.3	3.30×10^{6}
		5.0	3.8	1.8	14,916
		7.5	4.4	4.7	941
		10.0	5.0	9.5	188
		12.5	5.6	15.7	71
		15.0	6.0	22.8	39
		17.5	6.3	30.1	27
		20.0	6.5	37.2	22
		22.5	6.7	43.9	19
		25.0	6.8	50.2	17
		27.5	6.9	55.9	16
		30.0	7.0	61.1	15
		32.5	7.1	65.8	14
		35.0	7.2	69.9	14
		37.5	7.2	73.6	14
		40.0	7.2	76.9	13
10.0	E	2.5	5.5	0.0	2.50×10^{9}
		5.0	7.0	0.1	4.07×10^{6}
		7.5	10.4	1.2	18,352
		10.0	12.2	4.3	919
		12.5	12.9	9.9	157
		15.0	13.0	18.0	55
		17.5	13.0	27.6	30
		20.0	13.0	37.6	22
		22.5	13.0	46.9	18
		25.0	13.0	55.4	16
		27.5	13.0	62.8	14
		30.0	13.0	69.2	14
		32.5	13.0	74.7	13
		35.0	13.0	79.3	13
		37.5	13.0	83.2	13
		40.0	13.0	86.4	13

AQL	Sample size code letter	Incoming percent defective	Expected sample size	Expected percent lots rejected	Expected lots under sampling
10.0	F	2.5	8.1	0.0	1.48×10^{10}
		5.0	8.9	0.0	1.14×10^{9}
		7.5	13.8	0.2	410,188
		10.0	18.3	1.5	5,454
		12.5	19.7	5.0	415
		15.0	20.0	12.3	89
		17.5	20.0	23.9	37
		20.0	20.0	37.2	23
		22.5	20.0	49.9	18
		25.0	20.0	60.1	16
		27.5	20.0	69.8	14
		30.0	20.0	77.0	14
		32.5	20.0	82.6	13
		35.0	20.0	86.9	13
		37.5	20.0	90.3	13
		40.0	20.0	92.8	12
10.0	G	2.5	13.1	0.0	4.69×10^{10}
		5.0	13.8	0.0	3.94×10^{9}
		7.5	20.3	0.2	2.66×10^{6}
		10.0	29.7	1.6	8,562
		12.5	31.8	6.0	344
		15.0	32.0	16.5	60
		17.5	32.0	32.4	26
		20.0	32.0	48.7	18
		22.5	32.0	62.7	15
		25.0	32.0	73.8	14
		27.5	32.0	82.0	13
		30.0	32.0	88.0	13
		32.5	32.0	92.0	12
		35.0	32.0	94.8	12
		37.5	32.0	96.7	12
		40.0	32.0	97.9	12

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